Chin. J. Astron. Astrophys. Vol.0 (200x) No.0, 000–000
(*http://www.chjaa.org*)

Chinese Journal of
Astronomy and
Astrophysics

Testing Lorentz Violation Using Propagating UHECRs

Cong-Xin Qiu ⋆ and Zi-Gao Dai

Department of Astronomy, Nanjing University, Nanjing 210093, P. R. China

Abstract Lorentz invariant violation (LIV) test is very important to study in the new physics. All the known astrophysical constraints either have a very small examinable parameter space, or are only suitable for some special theoretical models. Here we suggest that it is possible to detect the time-delay of ultra-high-energy cosmic-rays (UHECRs) directly. We discuss some difficulties in our method, including the intergalactic magnetic fields. It seems that none of them are crucial, hence this method could give a larger examinable parameter space and a stronger constraint on LIV.

Key words: cosmic rays — gamma rays: bursts — ISM: magnetic fields — relativity

1 INTRODUCTION

Lorentz invariant violation (LIV) test (Pavlopoulos, 1967) is significant for the applicability of Special Relativity, even if it has few trustworthy theoretical foundations. The theoretical approaches for LIV are mainly from new physics, including the Standard-Model Extension, noncommutative geometry (Szabo, 2003), loop quantum gravity (Rovelli, 1998) and string theory (Green et al., 1987; Polchinski, 1998).

The Standard-Model Extension approach [1] (Colladay and Kostelecký, 1997, 1998) is the most straightforward one, which introduces LIV as an assumption. LIV may be caused by spontaneously violating the vacuum solution, if not by the theory itself. The minimum Standard-Model Extension wishes to maintain all the conventional desirable properties of the Standard-Model beside allowing for violations of Lorentz symmetry, hence it does not have many astrophysical (time-integral-kind) applications. However, the Standard-Model Extension can actually induce some kind of birefringence effects for photons (Kostelecký and Mewes, 2001).

Noncommutative geometry has a lot of phenomenological applications. However, most of them are based on terrestrial experiments (Hinchliffe et al., 2002; Konopka and Major, 2002). The derivation of *particle* Lorentz-violating terms from noncommutative geometry (Carroll et al., 2001) seems more natural than other approaches, but unfortunately, it does not have (at least we don't know how it can have) a beautiful and feasible way to be approved by time-integral-kind experiments. The differences are as follows. Some theoretical models result in a constant space of light (e.g. by κ-Minkowski space-time (Tamaki et al., 2002)). And, researchers seem to have different opinions on whether the Lorentz-violating term $\theta^{\mu\nu}$ depends or not on position, energy or momentum [2].

The loop quantum gravity approach seems the most usable one. The propagational calculations of photons (Alfaro et al., 2002a; Gambini and Pullin, 1999) and neutrinos (or other massive spin-$1/2$ fermions) (Alfaro et al., 2000, 2002b) are fulfilled. The propagation speeds in both cases are non-trivial, with velocity departure linearly depending on particle energy. Further more, photons have a first order and neutrinos have a second order birefringence effect. However, although "foam" structure (Doplicher et al., 1995; Garay, 1998; Hawking, 1978; Wheeler, 1964) is really an intuitive way to understand the nature of

⋆ E-mail: congxin.qiu@gmail.com

[1] <http://www.physics.indiana.edu/~kostelec/faq.html >

[2] The kind of experiments we are interested in work only if $\theta^{\mu\nu}$ depends on energy and momentum, but is independent or almost independent of position for us to integral the effect.

quantum space-time, we have to warn ourselves time and again that loop quantum gravity theory itself has some theoretical problems (Ashtekar et al., 1992), "weave" states and coarse graining approximation are at most effective models.

Another leading (and in fact, chronologically the "first") root for the LIV calculations is from Liouville string (Ellis et al., 1992), a phenomenological model which makes the calculations of propagation equation in the framework of string theory possible (Amelino-Camelia et al., 1997; Ellis et al., 2000a). What they can calculate are the so-called "photons" which are the endpoints of open-strings attached to D-branes, and the space-time foam is described by D-brane fluctuations. The model can result in Lorentz-violating propagation equation by LIV of string ground state, although it also has some inconsistencies. As a result, LIV is stochastic, and the degree of velocity departure is first ordered. However, there is no evidence to support birefringence, which is in conflict with loop quantum gravity results.

There are also some other ways to discuss LIV from the theoretical viewpoint, although some of them are formerly due to the so-called GZK anomaly, which may in fact be some kind of experimental errors (HiRes Collaboration, 2008; Pierre Auger Collaboration, 2007). The methods include simply adding tiny (first order or second order or whatever we want) Lorentz-violating terms to a conventional Lagrangian (these may be considered as some kind of Standard-Model Extension) and seeing how they can affect our observations (Coleman and Glashow, 1999; Myers and Pospelov, 2003), calculating the geodesic in a topological fluctuated *classical* general relativity to get some very complicated results (Yu and Ford, 1999), deforming the measure of integration in Feynman graphs (which is equivalent to inventing a new renormalization skill) to get an effective LIV (Alfaro, 2005a,b), calculating the graviton induced corrections to Maxwell's equations (Dalvit et al., 2001); however, the resultant speed of light correction in the last method is independent of energy. A recent work by Gogberashvili et al. (2007) deduces the dispersion relation (with no birefringence effect) from fat brane-world scenario; but the resultant constraint seems to be too strict to trust that model.

Astrophysical experimental (dis)confirmations of LIV often use far transient sources emitting high-energy particles. The common sources are gamma-ray bursts (GRBs) which are cosmological, have very short durations, and can emit high-energy photons (Gupta and Zhang, 2007) and neutrinos [3]. The other common sources are giant γ-ray flares of active galactic nuclei (AGNs); however, there have not been suitable models for the shapes of the time profiles until now. If energy can affect particle speed by the LIV effect (it's not the same as the effect of particle mass, which becomes unimportant if the particle is sufficiently energetic), as some theoretical works predicted, particles emit simultaneously from the source but with different energies will exhibit a time-delay when observed. The possible ways include testing the time-delay of prompt emission photons from GRBs (Amelino-Camelia et al., 1998; Ellis et al., 2000b, 2003, 2006; Norris et al., 1999) and giant γ-ray flares of AGNs (Biller et al., 1999; MAGIC Collaboration, 2007), the time-delay of neutrinos from GRBs (Alfaro et al., 2000, 2002b; Bertolami and Carvalho, 2000; Choubey and King, 2003; Jacob and Piran, 2007), the polarized photons from GRBs (Fan et al., 2007; Gleiser and Kozameh, 2001; G.Mitrofanov, 2003) and distant galaxies (Kostelecký and Mewes, 2001) which should be destroyed by birefringence (Alfaro et al., 2002a; Gambini and Pullin, 1999), the synchrotron radiation from the Crab nebula (Jacobson et al., 2002, 2003) which should not be observed if photons can be both superluminal and subluminal but electrons can only be subluminal (Myers and Pospelov, 2003). There are also a lot of theoretical works to explain the GZK anomaly by Lorentz-violating terms (Alfaro and Palma, 2003; Aloisio et al., 2000; Amelino-Camelia and Piran, 2001; Coleman and Glashow, 1999), so if the GZK cutoff (Greisen, 1966; Zatsepin and Kuz'min, 1966) does in fact exist, the inverse proportion may also give some kind of constraints.

The purpose of this paper is to suggest a different way to (dis)confirm the LIV effect; that is, to test directly the time-delay of ultra-high-energy cosmic-rays (UHECRs) from far away sources. This method may give a larger examinable parameter space and a stronger constraint.

---

[3] There are really a lot of different models for GRBs to emit ultra-high energy neutrinos, from (Waxman and Bahcall, 1997) until now. Nearly all of the scenarios are $p + p$ or $p + \gamma \Rightarrow \pi^+ \Rightarrow \nu$, but in different environments. See Waxman (2001a) for a review.

2 CALCULATION

2.1 Naive Time-Delays by the LIV Effect

One possible way to (dis)confirm LIV is simply to test the time-delay of UHECRs from far away sources. Because in mainstream quantum gravity models, the departure of velocities depends on energy *linearly* (Alfaro et al., 2000, 2002a,b; Amelino-Camelia et al., 1997; Ellis et al., 2000a; Gambini and Pullin, 1999) in the massless approximation, the time-delay is very sensitive to ultra-high-energy particles. A naive calculation shows that the time-delays are really huge. For example, in the standard cosmological model where H_0, $\Omega_{\rm m}$ and Ω_Λ as the customary cosmological parameters, the propagation equation and time-delay for a massless particle has the form

$$v = c\left(1 \pm \frac{E}{\xi E_{\rm pl}}\right) \tag{1}$$

and

$$\Delta t_{\rm QG} = \frac{1}{H_0}\int_0^z \left(\frac{E}{\xi E_{\rm pl}}\right)\frac{(1+z')dz'}{\sqrt{\Omega_{\rm m}(1+z')^3+\Omega_\Lambda}}, \tag{2}$$

where $E \ll E_{\rm pl}$ is the energy of the particle, z is the redshift of the source, ξ is a free parameter to describe the degree of violation (which we want to restrict) with assumed typical value of unity, c is the speed of light, and $E_{\rm pl}$ is the Planck energy. To give a straightforward example, insert $E = 10^{19.8}\,{\rm eV} \simeq 6.3\times10^{19}\,{\rm eV}$ as the GZK threshold energy, $z = 0.1 \simeq 400\,{\rm Mpc}$ as a nearby source, and $\xi = 1$ as a typical dimensionless free parameter, we have

$$\Delta t_{\rm QG} \simeq 7\,{\rm yr}. \tag{3}$$

We choose $z = 0.1 \simeq 400\,{\rm Mpc}$ rather than larger distances to avoid $\Delta t_{\rm QG}$ to be too large to be compared with human longevity. In this case, cosmological models are in fact irrespective, so the situation differs from considering less energetic but neutral particles (like photons or neutrinos) that come from more far away sources. Closer sources are also possible (and maybe even better), because nearly all the time-delay effects (including the intergalactic magnetic fields, which we discuss in detail in § 2.4) caused by propagation depend *linearly* on distance, and distance is irrespective when contrasting which one of the time-delay effects is more important. Remote sources are only needed when $\Delta t_{\rm QG}$ is too small compared with the internal duration of the events themselves, which is only several seconds for GRBs and some other transient sources.

When the energy of the UHECR particles exceeds the GZK threshold, it is less possible for the source to be too far away, because the particles lose energy by interacting with CMB photons. The main mechanisms of energy loss on the road are photomeson production (Stecker, 1968) and e^+e^- pair production (Blumenthal, 1970), with their mean free paths already being calculated. However, for the UHECR events with energy larger than the GZK threshold we have *already* observed, their time-delays by the LIV effect are really interesting, because they should be more energetic and more sensitive to LIV when just be emitted.

However, the calculation of energy loss rate dE/dx (where x is the propagation distance) is very difficult, although what we have to face are trivial details of standard quantum field theory and phase space integrals. Here we simply use the existing numerical results (Aharonian and Cronin, 1994; Cronin, 1992) to proceed our calculations. Time-delays depending on different propagation distances are shown in Fig. (1, bottom). We see that although the time-delays finally tend to the same level as others (because all their energies converge to the GZK threshold energy after a long way of propagation), their differences are tremendous when just been emitted. So, confirming the sources of the UHECR events above the GZK threshold energy may also be a way to test the LIV effect, even if the source is really nearby.

We have to emphasize here that the particle energy versus propagation distance relation in Fig. (1, top) is based on some statistical results with large samples, because the photomeson interaction is stochastic (see more detailed discussion in § 2.4.2). The mean free paths for UHECRs with energy $E \gtrsim 10^{21}\,{\rm eV}$ are about $10\,{\rm Mpc}$, and larger for less energetic ones (Stecker, 1968). A certain particle can only suffer from a couple of collisions before being observed, so the LIV constraint by one single UHECR event has its initial

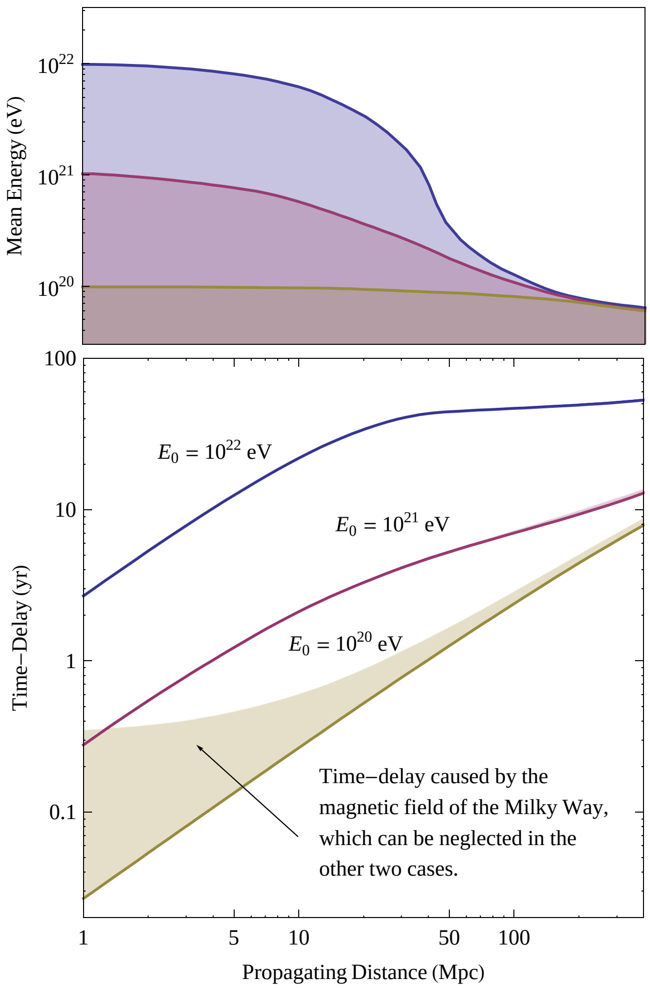

Fig. 1 Top, how particle energies change with propagation distance, if their initial energies are above the GZK threshold (Cronin, 1992). The initial energies are chosen to be 10^{20}, 10^{21} and $10^{22}\,\mathrm{eV}$. Bottom, the total time-delays caused by the LIV effect. The shadow regions are the time-delays caused by the magnetic field of the Milky Way, which can be neglectable undoubtedly in the late two cases.

measurement errors. However, the observed time-delay can at least give an upper limit for LIV, and we can still improve our result by averaging more events or using more advanced statistical methods.

2.2 Problem of the Applicability of the Time-delay Equation

To investigate the capacity of the method we propose, there are several problems which have to be considered carefully. The first problem is whether the naive propagation equation can be used for UHECRs. Although we are not sure what the compositions of UHECRs are, air shower data exclude photons or electrons, and the existence (HiRes Collaboration, 2008; Pierre Auger Collaboration, 2007) of the GZK cutoff (Greisen, 1966; Zatsepin and Kuz'min, 1966) suggests in fact protons or heavier nuclei. Although we cannot rule out other possibilities like exotic particles, we will assume in the context they are protons which are compound and have finite rest mass (discussions with the assumption that they are heavier nu-

clei like Fe are analogous). Mass is not a serious problem at the energy scale of the GZK threshold [4]. For example, in the case of parameters using above in Eq. (3), the time-delay affected by proton mass is only $5 \times 10^{-6}\,\mathrm{s}$, much shorter than Δt_{QG} affect by the LIV effect. A more serious problem is the complexity of protons. We all know that proton is made up of three quarks, therefore using the LIV calculations for *elementary* particles to calculate it will not be justified. A detailed study of quantum chromodynamics (QCD) with Lorentz-noninvariant terms is needed; however, it will certainly be very difficult. We still use the same propagation equation by some scaling arguments (Coleman and Glashow, 1999), or by simply thinking its effect as an overall constant coefficient, just like that of turning off QCD [5]. For the reason that the LIV confirmation is still qualitatively rather than quantitatively at present, an overall constant can be neglected.

2.3 Problem of the Source

2.3.1 Time Bases

The second problem is how we can choose the time bases, thus compute the time-delays with a suitable zero point. Because Δt should be typically very long, as shown in Eq. (3), it is a serious problem to know when UHECRs should come if we change their energies (because Δt_{QG} depends on energy of the particle) or turn off the LIV effect (as in the classical limit of $E \to 0$). Comparisons of events with photons or other low energy massless particles (which are certainly much less energetic and can be taken for as in the $E \to 0$ limit; hence the LIV effect of it can be neglected) and other UHECRs (with energy different from each other) from the same source can scale the time-delay. The precondition is that we have confirmed the source, or that we are convinced of the fact that different signals come from the same source.

As the recent powerful evidence (Pierre Auger Collaboration, 2007) shows, UHECRs come from some *extragalactic* sources because they are anisotropic and correlated with the direction of the Super-galactic plane. In that case, the mainstream models for sources are AGNs (Ginzburg and Syrovatskii, 1964; Hillas, 1984) and GRBs (Vietri, 1995; Vietri et al., 2003; Waxman, 1995, 2004; Wick et al., 2004), but other sources distributing within the Super-galactic plane are also possible (if they are related to, e.g. galaxy formation or stellar formation, which is always true). The main mechanism is Fermi acceleration but in different environments.

As AGNs are lasting sources, we can hardly know very well *when* the source emitted the UHECR particle we observed. However, some recent theoretical work (Farrar and Gruzinov, 2008) shows that the UHECR emissions are associated with AGN giant flares, with typical wait-time about 10^3 to 10^4 years (Donley et al., 2002). Because the duration is much longer than the typical time-delay we gave in Eq. (3), AGNs can be used in this method if the theoretical work mentioned above is true. GRBs are much better sources, because nearly all of the mainstream central engine models (including collapsars, supranovae and mergers of compact objects) tell us they are transient and burst only one time in their whole lives. If the emission of UHECRs and the burst itself happen almost at the same time (which is the most reasonable assumption), we can scale the time-delay by the observed low energy γ-rays because they can hardly be affected by mass, electromagnetic fields and the LIV effect. Other sources are also possible, if they emit particles (photons for instance; however, they are not exclusive choices) other than UHECRs which can be observed by our scientific equipments.

Are these kind of sources practicable for our purpose? The distances of the sources mentioned above are all suitable for the constraint that Δt_{QG} given in § 2.1 should not be too large. Short GRBs are often not too far away from us, and there are already a number of GRBs with redshift $z \sim 0.1$, including a special one (GRB 980425) with an especially small redshift $z = 0.0085$ (Galama et al., 1998). Although the number density of AGNs decreases quickly when $z < 1$, there are already hundreds of nearby AGNs have been observed until now (e.g. the V-C catalog (Véron-Cetty and Véron, 2006) has 694 AGNs with redshift $z \leq 0.024$). Similarly, it is reasonable to assume that other possible sources of UHECRs are not too far away, because UHECRs are not isotropically distributed in the celestial sphere.

---

[4] Of course, massive and massless particles are totally different from the viewpoint of quantum field theory. However, we give up discussing it deeply, because of the still inconsistent theoretical works.

[5] If we turn off QCD, the propagation equation can be used to every elementary particle inside a proton, so the overall effect is only a constant coefficient.

2.3.2 Confirmation of the UHECR Sources

The assumption in the above paragraph is that we have confirmed the source, or we are convinced of the fact that different signals come from the same source. However, it is not always the case. Notice the fact that UHECRs are singular events (it seldom happens that the UHECR events have clustering properties), confirm their sources by statistical correlation is very important.

Metrical bias in spatial dimensions are caused by (i) intergalactic magnetic fields and (ii) the uncertainties of detectors; the LIV effect cannot affect the orientation of UHECRs. If the collective effect of (i) and (ii) is small enough, we can confirm the sources by their locations in the celestial sphere; however, it may not be the case. If we assume that the effects of (i) and (ii) are both stochastic, confirmation of the sources is a pure statistical inferential problem. Astrophysical parameters only affect the statistical samples by (i) the UHECR energy band or (ii) the possible correlative time interval. Pierre Auger Collaboration (2007) has already discussed the statistical correlation between the arrival directions and the positions of known AGN. The same method can be used for our purpose; however, their arguments do not include the temporal dimension. When discussing the LIV effect, temporal dimension is very important. Hence, we should put by hand a possible correlative time interval when choosing the statistical samples; that is, assume that the collective time-delay caused by LIV, intergalactic magnetic fields and other reasons does not exceed this interval. Notice the fact that the observational history of UHECRs and correlative sources are at most several decades, which may be shorter than the collective time-delay, it is a good idea to ignore the temporal dimension and choose all the samples we know to do the statistical correlation. However, if the intergalactic magnetic fields are sufficiently large, we will never know the sources of UHECRs, no matter whether LIV exists or not.

2.4 Problem of the Intergalactic Magnetic Fields

The third but the most annoying problem is the intergalactic magnetic fields. Because protons take charges, their trajectories will be (Larmour) curved by magnetic fields, and the departures from straight lines will cause extra time-delays. Our method is only suitable when the time-delay $\Delta t_{\rm M}$ by the magnetic fields is less than by the LIV effect.

Because an UHECR particle keeps constant energy inside some homogeneous magnetic field, the time-delay should be

$$\Delta t_{\rm M} \simeq \frac{1}{24}\frac{D^3}{cr_{\rm L}^2} \simeq 0.79\left(\frac{D}{3\,{\rm kpc}}\right)^3\left(\frac{E}{6.3\times10^{19}\,{\rm eV}}\right)^{-2}\left(\frac{B_\perp}{1\,\mu{\rm G}}\right)^2\ {\rm yr}, \qquad (4)$$

where D is the linear distance of the trajectory, $B_\perp$ is the perpendicular magnitude of the magnetic field, E is the energy of the particle, and $r_{\rm L} = E/(c\cdot eB_\perp)$ is the Larmour radius.

2.4.1 Comparison with Photons

For simplicity, we first discuss the way of comparing the UHECRs' time-delay with photons, because photons are irrespective of the magnetic field, and their time-delay by the LIV effect can be neglected compared to UHECRs for their relatively lower energies.

The real trajectory can be devided into three parts, inside the host galaxy, inside our Galaxy and in the intergalactic media (IGM), that is, $\Delta t_{\rm M} = \Delta t_{\rm M,host} + \Delta t_{\rm M,Milky} + \Delta t_{\rm M,IGM}$. We have already chosen the values of D and $B_\perp$ both for a typical galaxy in Eq. (4), so $\Delta t_{\rm M,Milky} \sim 0.79\,{\rm yr}$ is the typical value for the time-delay effect of the Milky Way, which can be negligible compared to $\Delta t_{\rm QG}$ we estimated in Eq. (3). Of course, $\Delta t_{\rm QG}$ decreases when the source comes nearer, but the effect by the Milky Way's magnetic field remains unaltered, so it would be troublesome when considering the use of more nearby sources to test LIV, as mentioned in § 2.1. However, because the time-delay by magnetic fields is absolutely classical, when we have fine structure models for the magnetic field of our Galaxy someday, we can deduct this effect directly [6]. When the UHECR particles are initially more energetic than the GZK threshold, effect from the

[6] Because the correlation length of the magnetic field in our galaxy should be compared with the scale of the galaxy itself. Further more, we know the direction where the UHECR particle is related to the local structure.

Milky Way's magnetic field can always be negligible, as shown in Fig. (1, bottom). The time-delay by the host galaxy of the GRB will not be worse than by our Galaxy, because the energy E will be larger (if it formally exceeds the GZK threshold) or at less equal (if less than the GZK threshold) when just emitted.

However, the effect by the large from scale intergalactic magnetic fields is more thorny, because until now we lack good models for the magnitude and topological structure of the fields. A constraint from the CMB anisotropy (Barrow et al., 1997) gives

$$B_{\rm IGM} < 6.8 \times 10^{-9} (\Omega_0 h^2)^{1/2}\,{\rm G} \sim 4.9 \times 10^{-9}\,{\rm G}, \tag{5}$$

where we choose $\Omega_0 = 1$ and Hubble constant $H_0 = 72\,{\rm km\,s^{-1}\,Mpc^{-1}}$. Another constraint from the observed rotation measure (RM) of quasars (Kronberg, 1994) gives

$$B_{\rm IGM} < 10^{-9} \left(\frac{\lambda}{1\,{\rm Mpc}}\right)^{-1/2} {\rm G}, \tag{6}$$

where λ denotes the correlation length (coherence length) of the magnetic fields, as a reasonable assumption that the power spectrum of magnetic fields has a large scale cut-off.

If we assume that the field is conglomerated and homogeneous inside every segment (with typical scale of correlation length λ), the UHECR particle will randomly change its direction due to Larmour's motion, but goes a nearly strict line as a whole. If $B_{\rm IGM}$ is independent the correlation length λ, the overall time-delay should be

$$\Delta t_{\rm M,IGM} \simeq 1.18 \left(\frac{D}{400\,{\rm Mpc}}\right) \left(\frac{\lambda}{1\,{\rm Mpc}}\right)^2 \left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2} \left(\frac{B_\perp}{10^{-11}\,{\rm G}}\right)^2 {\rm yr}. \tag{7}$$

When B depends on the correlation length as $B = B_0 \lambda^{-1/2}\,{\rm G}$ in Eq. (6), the time-delay is

$$\Delta t_{\rm M,IGM} \simeq 1.18 \left(\frac{D}{400\,{\rm Mpc}}\right) \left(\frac{\lambda}{1\,{\rm Mpc}}\right) \left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2} \left(\frac{B_{0,\perp}}{10^{-11}\,{\rm G}}\right)^2 {\rm yr}. \tag{8}$$

Other possible parameters are denoted in Fig. (2). We see that it is needed [7] for our purpose that $B_{\rm IGM}$ is slightly less than the upper limits given by Eq. (5) and (6), unless we choose a smaller correlation length.

Things will be more interesting when consider the UHECR particles with energy exceeding the GZK threshold. Noting that $B_\perp$ ($B_{0,\perp}$) and λ in Eq. (7) and (8) are independent of the source properties, we may define

$$\eta \equiv 1.18 \left(\frac{\lambda}{1\,{\rm Mpc}}\right)^2 \left(\frac{B_\perp}{10^{-11}\,{\rm G}}\right)^2 \tag{9}$$

in Eq. (7) and

$$\eta \equiv 1.18 \times \left(\frac{\lambda}{1\,{\rm Mpc}}\right) \left(\frac{B_{0,\perp}}{10^{-11}\,{\rm G}}\right)^2 \tag{10}$$

in Eq. (8); then the effect by intergalactic magnetic field has a uniform expression

$$\Delta t_{\rm M,IGM} \simeq \eta \cdot \left(\frac{D}{400\,{\rm Mpc}}\right) \left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2} {\rm yr}. \tag{11}$$

In Fig. (3), we calculated $\Delta t_{\rm QG} + \Delta t_{\rm M,Milky} + \Delta t_{\rm M,IGM}$ in all, with $\eta = 1$, 70 and 5000 respectively. $\eta = 5000$ has already saturated the upper bound given by Eq. (5) and (6), so $\Delta t_{\rm M,IGM}$ cannot be larger. Noting that when $E_0 \geq 10^{21}\,{\rm eV}$, the UHECR particle will absolutely not be affected by magnetic fields if it is not too far away (roughly $D \leq 10\,{\rm Mpc}$), hence the *only* thing that can make a visible time-delay is the LIV effect. HiRes and AGASA have already observed a couple of the UHECR events with energy $E > 3\times 10^{20}\,{\rm eV}$ (AGASA Collaboration, 2003; HiRes Collaboration, 2005). If their distance $D > 20\,{\rm Mpc}$, their initial energy E_0 will exceed $10^{21}\,{\rm eV}$, as shown in Fig. (1, top). Hence, seeking the sources of UHECRs with energy $E > 3 \times 10^{20}\,{\rm eV}$ will tremendously help us to (dis)confirm the LIV effect.

[7] It is possible in principle because $B_{\rm IGM}$ remains largely unknown by the intrinsic observational difficulties (Beck et al., 1996).

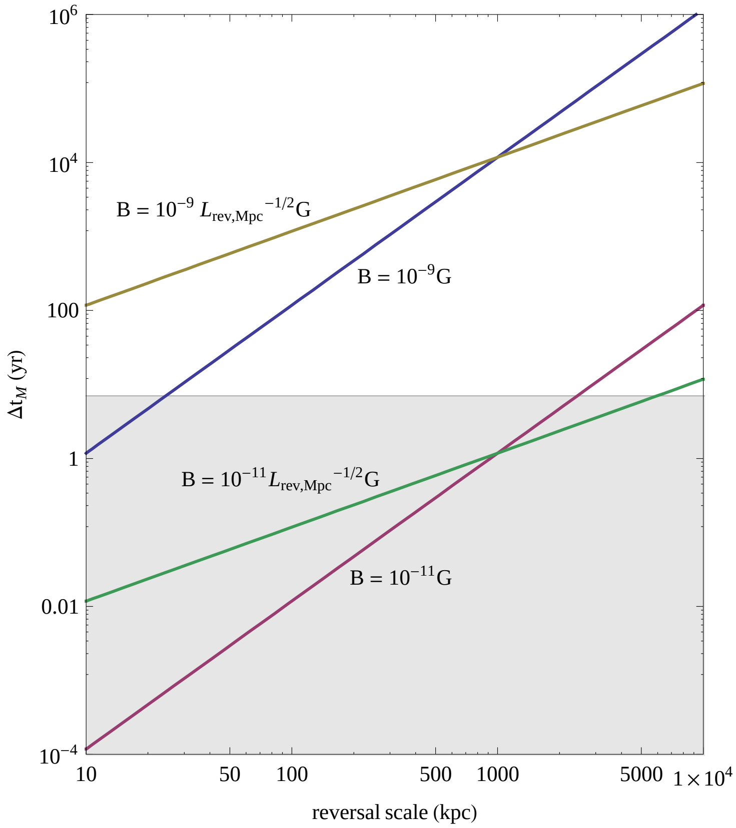

Fig. 2 Time-delay $\Delta t_{\rm M,IGM}$ caused by the intergalactic magnetic fields when $z = 0.1 \simeq 400\rm Mpc$ and $E = 6.3 \times 10^{19}\,\rm eV$, with different correlation length and strength of the magnetic fields. The horizontal line is $\Delta t_{\rm QG} = 7\,\rm yr$, as the example shows in Eq. (3). The method is useful when $\Delta t_{\rm M,IGM} < \Delta t_{\rm M} < \Delta t_{\rm QG}$.

Things will be worse if there exists a *global* cosmic magnetic field, or the fields have structures like filaments or sheets (Ryu, 1998). Those will cause larger time-delays. The first trouble can be easily seen from the expression of $\Delta t_{\rm M,IGM}$ given above, because it is equivalent to a huge λ. The second trouble is because, when a magnetic cloud collapses to $1/k$ of its diameter, the magnetic field strength will increase k^2 times than it used to be. Although the particle will miss a lot of clouds it used to bang into (only for the case of filaments but not sheets, which make things worse), we still have

$$B_\perp^2\lambda^2 \to \frac{1}{k}(k^2B_\perp)^2\left(\frac{1}{k}\lambda\right)^2 = kB_\perp^2\lambda^2, \tag{12}$$

and the same for the $B_\perp^2\lambda$ cases.

However, in fact the irrefutable observed anisotropy of UHECRs (Pierre Auger Collaboration, 2007) has already given us an upper limit for $B_\perp$ and λ, and also whether the magnetic field has already collapsed to filaments or sheets or not. Note that 20 among 28 highest energy events detected by the Pierre Auger Observatory are within a 3.1° circle of nearby AGNs (with distance less than $75\,\rm Mpc$ away). No matter whether we believe these UHECR particles to originate from those AGNs or not, it is unassailable that UHECRs are unisotropically distributed, and seem to be correlated to the Super-galactic plane. So, the angular dispersion caused by the intergalactic magnetic fields should be less than a couple of degrees. The

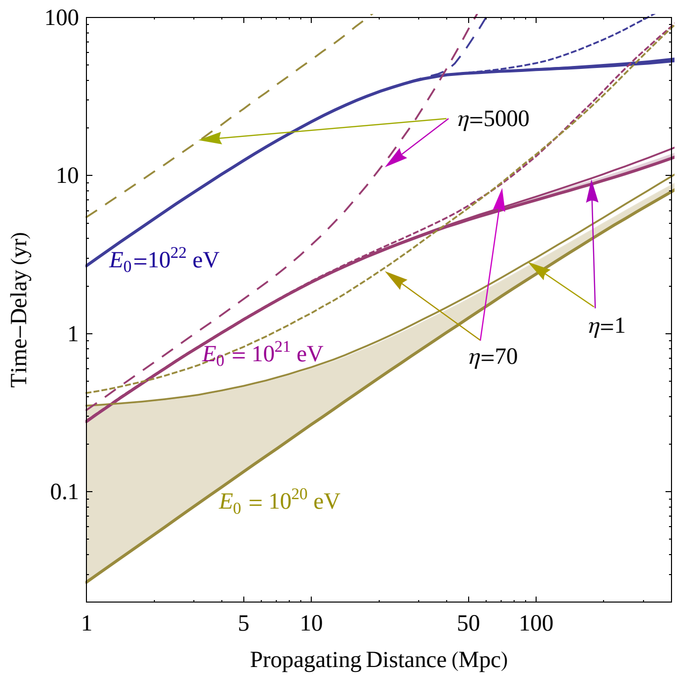

Fig. 3 The thick solid lines and shadow regions are the same as in Fig. (1). In addition, we calculated $\Delta t_{\rm QG} + \Delta t_{\rm M,Milky} + \Delta t_{\rm M,IGM}$ with $\eta = 1$, 70 and 5000 respectively. It seems that finding the sources of the UHECR events with energy $E > 3 \times 10^{20}$ eV will tremendously help us to (dis)confirm the LIV effect. See the context for detail.

angle departure inside some homogeneous field is

$$\alpha \simeq \frac{D}{2r_{\rm L}}, \tag{13}$$

and the different irrelevant magnetic bulks (with typical size λ) can be considered as a random walking process. The overall angle departure is

$$\alpha \simeq \frac{\lambda}{2r_{\rm L}}\sqrt{\frac{D}{\lambda}} = \frac{\sqrt{D\cdot\lambda}}{2r_{\rm L}}. \tag{14}$$

Choosing $D = 100\,{\rm Mpc}$ as the typical scale of Super-galactic plane, we have

$$\alpha_{\rm Milky} \simeq 1.26^\circ \left(\frac{D}{3\,{\rm kpc}}\right)\left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-1}\left(\frac{B_\perp}{10^{-6}\,{\rm G}}\right), \tag{15}$$

and

$$\alpha_{\rm IGM} \simeq 4.21^\circ \left(\frac{D}{100\,{\rm Mpc}}\right)^{1/2}\left(\frac{\lambda}{1\,{\rm Mpc}}\right)^{1/2}\left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-1}\left(\frac{B_\perp}{10^{-9}\,{\rm G}}\right). \tag{16}$$

Notice that when $\alpha_{\rm IGM}$ approaches a couple of degrees, as the above equation shows, the upper limits given by Eq. (5) and (6) have already been saturated. In addition, because of the fact that $\lambda^{1/2}B_\perp \to \sqrt{k}\lambda^{1/2}B_\perp$, filaments or sheets can also be suppressed.

One question is whether the Pierre Auger data tell us that $\alpha_{\rm IGM}$ should be equal to (rather than less than) several degrees? Absolutely not. $\alpha_{\rm IGM}$ can also be much less (so B and λ can also be much less).

Even if we have confirmed some UHECR sources, the angular dispersion can also be caused by reasons other than $\alpha_{\rm IGM}$, for example, magnetic field of the Milky way or simply the measurement errors.

We notice that some authors gave a larger $\Delta t_{\rm M,IGM}$ compared to ours given in Eq. (5) and (6). Waxman and Miralda-Escudé (1996) gave $\Delta t_{\rm M,IGM} \sim 100\,{\rm yr}$ because their correlation length $\lambda \sim 10\,{\rm Mpc}$ is 10 times larger than ours (equivalent to $\eta = 100$ in our definition). Sigl (2001) gave $\Delta t_{\rm M,IGM} \sim 10^3\,{\rm yr}$ because he chose a really large magnetic field $B \sim 10^9\,{\rm G}$ (equivalent to $\eta = 10^4$); however, with a smaller travelling distance D. Waxman (2001b) gave an *upper* bound of $\Delta t_{\rm M,IGM}$ even as large as $10^7\,{\rm yr}$, because his typical magnetic field $B \sim 10^8\,{\rm G}$ is really huge. He also argued that $\Delta t_{\rm M,IGM} > 100\,{\rm yr}$ by some statistical reasons of nearby source candidates and the UHECR events above the GZK threshold. The first two estimations are consistent with our constraint from correlation of Super-galactic plane and the UHECR events, the few discrepancies are just because we choose different typical parameters (which are all possible according to our current knowledge, because we know really little about the true value of B and λ) to write our formulas. We suggest that the anisotropy of UHECRs can give a tighter constraint of intergalactic magnetic field strength B, so the upper bound of $\Delta t_{\rm M,IGM}$ we can assure at present should be as low as $10^{4-5}\,{\rm yr}$. The *lower* bound $\Delta t_{\rm M,IGM} > 100\,{\rm yr}$ can be overcome because we know really little about both possible nearby sources and the UHECRs events, and the estimation is dependent on some details of source models. In addition, all the estimations given above are only suitable for particles with energy below the GZK threshold, because the energy loss is ignored. As we show in Fig. (3), the effect of intergalactic magnetic field is much less important if the energy of the *observed* UHECR event is much larger than the GZK threshold.

2.4.2 Comparison with Other UHECR Events

We can also compare the time-delay with other UHECR events (with slightly different energies), emitted nearly simultaneously from the same source. Of course, because the UHECR events are really rare, it may hardly happen.

In this case, blurs in both arrival direction and time-delay have to be analyzed carefully. (i) Blurs have two reasons. Particles with different energy follow different trajectories, thus leading to different directions and time-delays, because of the random topological distributions of the intergalactic magnetic fields. (ii) At the same time, particles above the GZK threshold energy would interact with CMB photons as the Poisson processes, introducing extra randomicity. Waxman and Miralda-Escudé (1996) discussed the blur effect with UHECRs below the GZK threshold, in which case energy loss by photomeson production can be ignored. At the end of § 2.1, we have already discussed a little the influence of the LIV time-delay by stochastic photomeson production.

It is easy to understand that when the particles are extremely energetic, blurs in both arrival direction and time-delay caused by the intergalactic magnetic fields become less important. However, using one of the UHECRs to scale the others may be dangerous, if their energies are large enough (e.g. larger or equal to the GZK threshold) to make us believe that they have suffered one or more times the photomeson interactions. Because of the randomicity of Poisson arrival photomeson interactions, particles observed with the same energy from the same source may have tremendously different interacting histories and thus possess different time-delays caused by both the intergalactic magnetic fields and the LIV effects.

However, for UHECRs less energetic than the GZK threshold, photomeson production is turned off, and comparison becomes possible. The requirement that the intergalactic magnetic fields should not be very large, is the same as in the case of comparing UHECRs with photons, which we have already discussed in § 2.4.1.

2.5 Problem of the Energy Measurements in Air Shower Detectors

Notice that the energy measurements in different mass composition of the UHECR events and different air shower detectors have disagreements from each other which cannot be negligible, so it is necessary to discuss here the influence of the LIV confirmation by energy demarcation uncertainties. In § 2.4, we have already discussed two different methods to restrict LIV, the comparison (i) with photons and (ii) with the different UHECRs respectively from the same source.

For the reason that the investigations of the LIV confirmation are qualitatively rather than quantitatively at present, the energy metrical uncertainties are not crucial for method (i), because it can only introduce an order one coefficient of ξ in Eq. (2). When the UHECR events are not too energetic to neglect the time-delay caused by the intergalactic magnetic fields, *absolute* energy measurements are important. However, a global constraint for the collective influence of $\Delta t_{\rm QG} + \Delta t_{\rm M,IGM}$, hence the upper limits for both $\Delta t_{\rm QG}$ and $\Delta t_{\rm M,IGM}$ respectively, are still suitable for our purpose.

For method (ii), things are a little more complicated. Uncertainties introduced by the different assumptions of mass composition are not crucial. The reason is that, when assuming different UHECRs to be the same kind of particles (protons in our context), a mistaken assumptive mass composition can only introduce an order one coefficient of ξ, just as in the case of method (i). However, it is intractable for UHECRs detected by different air shower detectors with different energy metrical techniques. A wiser way is to choose some kind of calorimetric measurements to determine UHECRs' energies by different detectors (like fluorescence light emissions (Linsley, 1983; Song et al., 2000)) which are relatively model independent. As a matter of fact the discussions of LIV are presently still superficial, we may hope that energy demarcations are finer for further investigations of LIV in the near future.

3 DISCUSSION

3.1 Two Known Events

There was an archaeological report about the association of UHECRs and GRBs (Milgrom and Usov, 1995). The authors found that GRB 910503 and 921230 are associated with two highest-energy cosmic-ray shower events, with really small error boxes and time-delays of 5.5 and 11 months respectively. If GRBs are really sources for those two UHECR events, there are very strong constraints both for LIV and the strength of intergalactic magnetic fields (as the time-delay is much shorter than the naive estimation we make in Eq. (3)), because all effects such as rest mass, magnetic fields and quantum gravity, are addible, and to ignore some of them gives the upper constraint for the rest ones. However, we should not be too serious for that kind of stories, because they may only be a coincidence.

3.2 Comparison with Other Models

Although there are other constraints of LIV which are much stronger than the method we suggested, the method mentioned above has also its special purpose. Birefringence (Fan et al., 2007; Gleiser and Kozameh, 2001; G.Mitrofanov, 2003) can only be calculated in the framework of loop quantum gravity but not in other approaches, hence it may be wrong in a whole. The synchrotron radiative constraint (Jacobson et al., 2002, 2003) depends on a special theory (Myers and Pospelov, 2003), which needs a dimension-5 Lorentz-violating terms to induce birefringent photons but subluminal electrons (whose maximum speed cannot converge to c). The inverse proportion of the GZK anomaly may also give some stronger constraint. However, the scattering dynamical discussions are always only one-sided, which means that a scattering channel is open or suppressed only if the effect of LIV in opposite for two relative particles (and therefore their velocity as well as their effective mass being different). Although in the old days, the GZK anomaly is the most important reason for theoreticians to study LIV, its inexistence (HiRes Collaboration, 2008; Pierre Auger Collaboration, 2007) has not borne down the LIV subjects.

Testing the time-delay of UHECRs is a more direct way to study LIV. It can contain most kinds of theoretical works. If the intergalactic magnetic fields are sufficiently small (which is still absolutely consistent with the observations until now), it may have larger examinable parameter space for violation scale ξ (in Eq. (2)) than using photons or neutrinos. Even if its examinable parameter space is in fact much smaller, for the reasons mentioned above, the other causations are all classical and thus can someday be subtracted by models.

4 CONCLUSIONS

We have suggested to (dis)confirm LIV by simply detecting the time-delay of UHECRs. We considered some other reasons which also cause the time-delay, including the intergalactic magnetic fields. If the en-

ergy of the UHECR events we observed is *below* the GZK threshold $6.3 \times 10^{19}\,\mathrm{eV}$, a typical intergalactic magnetic field $B \lesssim 10^{-11}\,\mathrm{G}$ and correlation length $\lambda \lesssim 1\,\mathrm{Mpc}$ may be needed to give a parameter space examinable enough to constrain LIV. However, for an UHECR event with energy larger than $3 \times 10^{20}\,\mathrm{eV}$, our method is always possible. Because of the fact that we know really little about the intergalactic magnetic field's strength, if it is much smaller than the current upper limit $B \lesssim 10^{-9}\,\mathrm{G}$, our method may give a larger examinable parameter space and a stronger constraint of LIV than other constraints.

ACKNOWLEDGEMENTS

We would like to thank Yi-Zhong Fan and Xiang-Yu Wang for helpful discussions. This work is supported by the National Natural Science Foundation of China (grants 10221001 and 10640420144) and the National Basic Research Program of China (973 program) No. 2007CB815404.

References

AGASA Collaboration, 2003. Astropart. Phys. 19, 447.
Aharonian, F. A., Cronin, J. W., 1994. Phys. Rev. D 50, 1892.
Alfaro, J., 2005a. Phys. Rev. Lett. 94, 221302.
Alfaro, J., 2005b. Phys. Rev. D 72, 024027.
Alfaro, J., Morales-Técotl, H. A., Urrutia, L. F., 2000. Phys. Rev. Lett. 84, 2318.
Alfaro, J., Morales-Técotl, H. A., Urrutia, L. F., 2002a. Phys. Rev. D 65, 103509.
Alfaro, J., Morales-Técotl, H. A., Urrutia, L. F., 2002b. Phys. Rev. D 66, 124006.
Alfaro, J., Palma, G., 2003. Phys. Rev. D 67, 083003.
Aloisio, R., et al., 2000. Phys. Rev. D 62, 053010.
Amelino-Camelia, G., Piran, T., 2001. Phys. Rev. D 64, 036005.
Amelino-Camelia, G., et al., 1997. Int. J. Mod. Phys. A 12, 607.
Amelino-Camelia, G., et al., 1998. Nature 393, 763.
Ashtekar, A., Rovelli, C., Smolin, L., 1992. Phys. Rev. Lett. 69, 237.
Barrow, J. D., Ferreira, P. G., Silk, J., 1997. Phys. Rev. Lett. 78, 3610.
Beck, R., et al., 1996. ARA&A, 155.
Bertolami, O., Carvalho, C. S., 2000. Phys. Rev. D 61, 103002.
Biller, S. D., et al., 1999. Phys. Rev. Lett. 83, 2108.
Blumenthal, G. R., 1970. Phys. Rev. D 1, 1596.
Carroll, S. M., et al., 2001. Phys. Rev. Lett. 87, 141601.
Choubey, S., King, S. F., 2003. Phys. Rev. D 67, 073005.
Coleman, S., Glashow, S. L., 1999. Phys. Rev. D 59, 116008.
Colladay, D., Kostelecký, V. A., 1997. Phys. Rev. D 55, 6760.
Colladay, D., Kostelecký, V. A., 1998. Phys. Rev. D 58, 116002.
Cronin, J. W., 1992. Nucl. Phys. B (Proc. Suppl.) 28, 213.
Dalvit, D. A. R., Mazzitelli, F. D., Molina-París, C., 2001. Phys. Rev. D 63, 084023.
Donley, J. L., Brandt, W. N., Eracleous, M., Boller, T., 2002. AJ 124, 1308.
Doplicher, S., Fredenhagen, K., Roberts, J. E., 1995. Commun. Math. Phys. 172, 187.
Ellis, J., Mavromatos, N., Nanopoulos, D., 1992. Phys. Lett. B 293, 37.
Ellis, J., Mavromatos, N. E., Nanopoulos, D. V., 2000a. Gen. Rel. Grav. 32, 127.
Ellis, J., et al., 2000b. ApJ 535, 139.
Ellis, J., et al., 2003. A&A 402, 409.
Ellis, J., et al., 2006. Astropart. Phys. 25, 402.
Fan, Y.-Z., Wei, D.-M., Xu, D., 2007. MNRAS 376, 1857.
Farrar, G. R., Gruzinov, A., 2008. Preprint (0802.1074).
Galama, T. J., et al., 1998. Nature (London) 395, 670.
Gambini, R., Pullin, J., 1999. Phys. Rev. D 59, 124021.
Garay, L. J., 1998. Phys. Rev. Lett. 80, 2508.
Ginzburg, V. L., Syrovatskii, S. I., 1964. The Origin of Cosmic Rays. Pergamon, Oxford.
Gleiser, R. J., Kozameh, C. N., 2001. Phys. Rev. D 64, 083007.

G.Mitrofanov, I., 2003. Nature 426, 139.
Gogberashvili, M., Sakharov, A. S., Sarkisyan, E. K., 2007. Phys. Lett. B 644, 179.
Green, M., Schwarz, J., Witten, E., 1987. Superstring Theory. Cambridge University Press.
Greisen, K., 1966. Phys. Rev. Lett. 16, 748.
Gupta, N., Zhang, B., 2007. MNRAS 380, 78.
Hawking, S. W., 1978. Nucl. Phys. B 144, 349.
Hillas, A. M., 1984. ARA&A 22, 425.
Hinchliffe, I., Kersting, N., Ma, Y. L., 2002. Int. J. Mod. Phys. A 19, 179.
HiRes Collaboration, 2005. Astropart. Phys. 23, 157.
HiRes Collaboration, 2008. Phys. Rev. Lett. 100, 101101.
Jacob, U., Piran, T., 2007. Nature Phys. 3, 87.
Jacobson, T., Liberati, S., Mattingly, D., 2002. Phys. Rev. D 66, 081302.
Jacobson, T., Liberati, S., Mattingly, D., 2003. Nature 424, 1019.
Konopka, T. J., Major, S. A., 2002. New J. Phys. 4, 57.1.
Kostelecký, V. A., Mewes, M., 2001. Phys. Rev. Lett. 87, 251304.
Kronberg, P. P., 1994. Rep. Prog. Phys., 325.
Linsley, J., 1983. Rapporteur's talk given at 18th Int. Cosmic Ray Conf., Bangalore, India, Aug 22-Sep 3.
MAGIC Collaboration, 2007. Preprint (0708.2889).
Milgrom, M., Usov, V., 1995. ApJ L449, 37.
Myers, R. C., Pospelov, M., 2003. Phys. Rev. Lett. 90, 211601.
Norris, J., et al., 1999. Bulletin of the American Astronomical Society 31, 717.
Pavlopoulos, T. G., 1967. Phys. Rev. 159, 1106.
Pierre Auger Collaboration, 2007. Science 318, 938.
Polchinski, J., 1998. String Theory. Cambridge University Press.
Rovelli, C., 1998. Loop quantum gravity. Living Reviews in Relativity.
Ryu, D., 1998. A&A 335, 19.
Sigl, G., 2001. In: Lemoine, M., Sigl, G. (Eds.), Physics and Astrophysics of Ultra-High-Energy Cosmic Rays. Springer, p. 197.
Song, C., et al., 2000. Astropart. Phys. 14, 7.
Stecker, F. W., 1968. Phys. Rev. Lett. 21, 1016.
Szabo, R. J., 2003. Phys. Rept. 378, 207.
Tamaki, T., et al., 2002. Phys. Rev. D 65, 083003.
Véron-Cetty, M.-P., Véron, P., 2006. A&A 455, 773.
Vietri, M., 1995. ApJ 453, 883.
Vietri, M., Marco, D. D., Guetta, D., 2003. ApJ L594, 32.
Waxman, E., 1995. Phys. Rev. Lett. 75, 386.
Waxman, E., 2001a. Nucl. Phys. B (Proc. Suppl.) 91, 494.
Waxman, E., 2001b. In: Lemoine, M., Sigl, G. (Eds.), Physics and Astrophysics of Ultra-High-Energy Cosmic Rays. Springer, p. 122.
Waxman, E., 2004. ApJ 606, 988.
Waxman, E., Bahcall, J., 1997. Phys. Rev. Lett. 78, 2292.
Waxman, E., Miralda-Escudé, J., 1996. ApJ 472, L89.
Wheeler, J. A., 1964. In: DeWitt, C., DeWitt, B. S. (Eds.), Relativity, Groups and Topology. Gordon and Breach.
Wick, S. D., Dermer, C. D., Atoyan, A., 2004. Astroparticle Phys. 21, 125.
Yu, H., Ford, L. H., 1999. Phys. Rev. D 60, 084023.
Zatsepin, G. T., Kuz'min, V. A., 1966. JETP L4, 78.